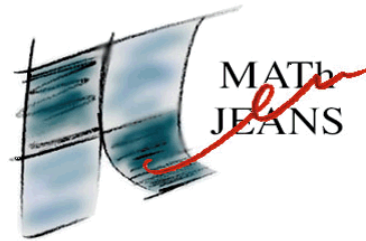




Erasmus+



Atelier Graines de chercheurs 2017-2018  
Erasmus+ project Mathematics and languages :  
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Subject 1: To each his own mania

You are fond of counting, it is almost a mania. When you see a number, you add its digits up and, if this sum divides the number you started with, you feel good! How often does it happen?

Subject 2: A bright idea

The walls of a room are all mirrors. A ray of light is sent parallel to the floor from a given point in the room. Which possible trajectories of the light can we observe depending on the original direction and the shape of the room?

Subject 3: Needle thread

If some day you are bored try this experience: take a needle and drop it onto the floor. If the needle lands on a line between two strips you win, if not you lose (of course we consider a floor made of parallel strips of wood of the same width, and a needle shorter than the width of each floorboard). Repeat this experience until you are tired and, eventually calculate your success rate. Which value can we expect for that rate if we realize the experiment a large amount of times? Why may the result be interesting?

Subject 4: Fractions with a nice profile

A positive fraction is said to have a nice profile if it can be written as a sum of other fractions, all different, each one of the form  $\frac{1}{p}$  with  $p$  a positive integer.

For instance, the fraction  $\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$  has a nice profile.

There are a lot of questions that can be asked about that topic. For example, which fractions have a nice profile? Is it possible to find general and automatic methods to write any given fraction in this way? If a fraction has a nice profile, how many ways are there to break it down? What if we are interested in sums of two fractions only? etc...

Subject 5: Take the fold!

What kind of mathematical object is it possible to obtain using folds (forms, numbers, curves, proofs, etc.)?

Subject 6: Very hot!

A big Christmas party is organized. Every guest must bring a gift and leave it into a big basket when they arrive. Around midnight, Santa offers a present from the basket randomly to each guest. Ideally, everyone should get a present different from the one they brought (we assume that all the gifts are different). What is the chance that it actually happens?

Subject 7: A special sequence

A sequence of integers is defined by  $a_1 = 2, a_2 = 3$  and, for each  $n \geq 2$ :

either  $a_{n+1} = 2a_{n-1}$  or  $a_{n+1} = 2a_n - 3a_{n-1}$ .

Can that sequence reach any of the following numbers: 17 ? 21 ? 1600? 1536 ? 2017 ?

Subject 8: A colored polygon

A convex  $N$ -gone is divided into triangles by its diagonals (assuming that 3 diagonals never intersect in the same point). The triangles are colored in red and in blue so that two triangles with a common side always have different colors. Find, in terms of  $N$ , the greatest possible value of the difference between the amount of red triangles and the amount of blue ones.

Subject 9: We get 2017

At the beginning you only have an empty blackboard. At each step you can either write twice the number 1 on the blackboard or delete two numbers equal to  $n$  already written and replace them by  $n - 1$  and  $n + 1$ .

How many steps at least will be necessary in order to reach the number 2017?